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The Application of Kiuttu's Formulation to Study Coaxial Flux Compression Generators *

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Abstract

A class of flux compression generators (FCGs) is based on the compression of the cross-sectional area of a coaxial geometry where the current flows along the outer conductor and returns through the inner conductor. This compression causes an increase in current since magnetic flux must be conserved. Kiuttu's inductive electric field formulation is a powerful tool for the conceptual design of coaxial FCGs [1]. The usefulness of this formulation is demonstrated in this paper for a simplified geometry using a finite element partial differential equation solver, FlexPDE ™, for calculation of the inductive electric field. A time varying applied current or a moving surface creates the non-conservative electric field. Losses due to diffusion of magnetic flux into conducting surfaces can also be accounted for and modeled in this setting. This analytical-computational approach serves as an important step in validating the MHD portion of the complex multi-physics parallel LLNL code, ALE3D. The non-intuitive boundary conditions involved in solving the otherwise straightforward partial differential equations are described in detail and illustrated in a simple model. The physical parameters used in the simulations are not based on a specific design.

I. INTRODUCTION

In Kiuttu's formulation[1], inductive electric field in pulsed coaxial devices where $B = B_\theta = \mu_o I/(2\pi r)$ is calculated by solving a Poisson-like PDE,

$$\nabla^2(\vec{F}) = -\varepsilon \frac{\partial \vec{B}}{\partial t} = -\frac{\varepsilon_r \dot{I}}{2\pi r\ c^2}\hat{\theta} \qquad (1)$$

where electric vector potential $\vec{F}$ is defined such that,

$$\vec{E} = -\frac{\nabla \times \vec{F}}{\varepsilon} \qquad (2)$$

with a "Coulomb-type" gauge condition for $\vec{F}$

$$\nabla \cdot \vec{F} = 0 \qquad (3)$$

Note that when viewed in cylindrical coordinates, $\vec{F}$ has only $\hat{\theta}$ dependence, whereas electric field $\vec{E}$ has no $\hat{\theta}$ dependence,

$$\vec{E} = \left[\frac{1}{\varepsilon}\frac{\partial F_\theta}{\partial z},\ 0\ , -\frac{1}{\varepsilon}\frac{\partial (rF_\theta)}{r\ \partial r}\right] \qquad (4)$$

To address the boundary conditions for the PDE in Equation (1), it is best to view $\vec{E}$ and $\vec{F}$ in terms of their normal and parallel components. Equation (2) can be rewritten as,

$$\vec{E} = \frac{1}{\varepsilon}\vec{\nabla} \times \vec{F} = \frac{1}{\varepsilon}\left(\vec{\nabla}_\perp + \vec{\nabla}_\parallel\right) \times \vec{F} \qquad (5)$$

From Equation (5), it follows that;

$$\nabla_\perp F = -\varepsilon E_\parallel \qquad (6a)$$

$$\nabla_\parallel F = \varepsilon E_\perp \qquad (6b)$$

Equation (6a) and Equation (6b) establish the boundary conditon for the problem. Equation (1) is then solved with a finite element PDE solver such as FlexPDE[2] or COMSOL[3] with the preceeding boundary conditions. In above equations, ε_r is the relative permittivity in sub-regions and is assumed to be uniform, $\varepsilon = \varepsilon_r\, \varepsilon_o$ and c is the speed of light. $\dot{I}$ is the rate of change of current and r is the radial distance from the center-line in an azimuthally symmetric geometry. Notations $\parallel$ and $\perp$ refer to parallel and perpendicular component of the vector.

II. BOUNDARY CONDITIONS

The correct treatment of boundary conditions as defined in Equation (6) is the most challenging part of solving Equation (1). To borrow terms from the FlexPDE manual, the boundary conditions for this PDE are either *Natural* or *Value*. Equation (6a) is of *Natural* form and Equation (6b) is of *Value* form. As will be shown, a moving boundary or/and resistive boundary constitutes a *Natural* boundary condition. On the other hand, *Value(F)* is a Dirichlet boundary that is applied only once throughout the problem to find a converged solution.

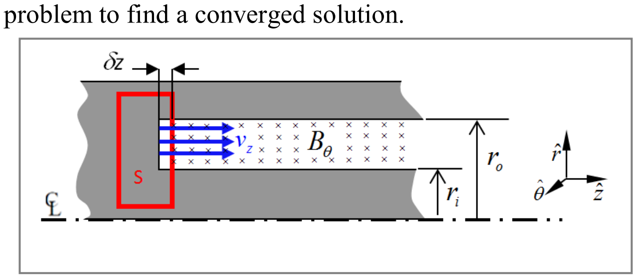

Figure 1: A moving conducting boundary in a seed magnetic field generates an *emf.*

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A. *Natural(F)*

There are two types of *Natural* boundary conditions; 1) a moving boundary or/and 2) a resistive boundary. Both types of BCs can co-exist on the same boundary.

1) Case of a Moving Boundary

In Figure 1, the movement of the left-boundary in +z in the seed magnetic field B_θ generates an *emf*. One can write,

$$emf = \frac{1}{\varepsilon}\int\left(\vec{n}\cdot\nabla F\right)dl = -\frac{1}{\varepsilon}\int_{r_i}^{r_o}\left(\vec{n}\cdot\nabla F\right)dr$$
$$= -\frac{\partial}{\partial t}\iint_S B_\theta\, dz\, dr = \lim_{\delta z\to 0} -\int_{r_i}^{r_o}\left(\frac{\partial B_\theta}{\partial t}\delta z + B_\theta v_z\right)dr = I\dot{L} \tag{7}$$

where $B_\theta = B$ is the background magnetic field, μ_o is the permeability of free space, $\vec{n}$ is the unit vector normal to the moving edge, r_i is the inner radius of the moving edge, r_o is the outer radius of the moving edge. I is the current, v_z is the wall velocity, and $\dot{L}$ is the change in inductance. From Equation (5),

$$Natural(F) = \vec{n}\cdot\nabla F = \varepsilon v_z B \tag{8}$$

Note also that a moving wall, induces a change in the seed magnetic field, this in turn invokes the Faraday Law,

$$\iint_S\left(\nabla\times\vec{E}\right)\cdot d\vec{A} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_S B\, dA = -I\dot{L} \tag{9}$$

where L is vacuum inductance of the coax defined as,

$$L = \frac{\mu_o l}{2\pi}\ln\left(\frac{r_o}{r_i}\right) \tag{10}$$

Note that the algebraic sum of Equation (7) and Equation (9) yields the lossless generator equation:

$$\frac{\partial(LI)}{\partial t} = 0 \tag{11}$$

2) Case of a Resistive Boundary

A resistive boundary is another form of the *Natural* boundary condition. In this case, the induced *emf* is generated from diffusion of magnetic field into the conductor, i.e., the IR loss,

$$Natural(F) = \nabla_\perp F = -\varepsilon E_\parallel$$
$$= -\varepsilon I\frac{\rho}{2\pi\,\delta r} = -\varepsilon I\frac{\Omega}{r} \tag{12}$$

where, $$\Omega = \frac{\rho}{2\pi\,\delta} \tag{13}$$

and , $$R = \int\frac{\Omega}{r}dl \tag{14}$$

In these expressions, ρ is the resistivity, δ is the magnetic skin depth and R is the resistance. As a special case, for perfectly electric conductors i.e., PEC, $Natural(F) = 0$

B. *Value(F)*

A region on the boundary where there is no $E_\perp$ constitutes a *Value(F)* boundary condition. Consider Equation (1), for a given ε and $\dot{I}$, the solution of the PDE places the upper limit of the value on the right-hand side of the equation for numerical precision,

$$Value(F) \le -\frac{\varepsilon\dot{I}}{2\pi r\ c^2} = \frac{Const.}{r} \tag{15}$$

To simplify, let $Const. = 0$

$$Value(F) = 0 \tag{16}$$

Again, this boundary condition is to be applied to region where $E_\perp \approx 0$. To obtain a converged solution the *Value(F)* boundary condition is applied only once to a boundary or portion of it. The choice in location may not be obvious at first – see Examples.

III. EXAMPLES

The usefulness of this technique is best illustrated in the following examples. For brevity, *Natural(F)* is written as *N(F)* and *Value(F)* as *V(F)* from this point forward and throughout this paper $B = B_\theta = \mu_o I/(2\pi r)$.

A. *Example 1: Parallel Field Drive and PEC Walls*

We start with a coaxial geometry (r_o = 3.5 cm, r_i = 2.5 cm and l = 10.0 cm), ε =1. All walls are made of PEC and a centrally placed 45° insulator (ε =5) in the channel. The insulaor is 3.0 cm away from the left side-wall of the channel as shown in Figure 2. The vacuum inductance of this geometry, using Equation (10), is calculated to be 6.2729 nH. If we were to inject a varying in time current $\dot{I}$ across nodes 1 and 2, then the *emf* induced across the two nodes will be $\dot{I}L$ which is realized by setting the *V(F)=0* constraint on the left-hand side boundary.

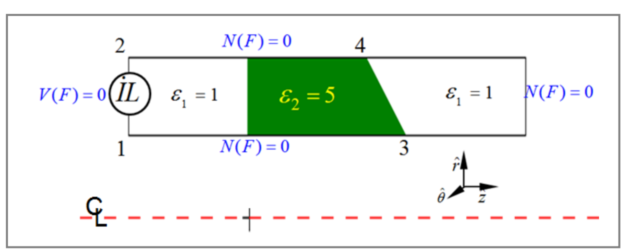

Figure 2. A coaxial geometry with parallel field drive and PEC walls; example 1.

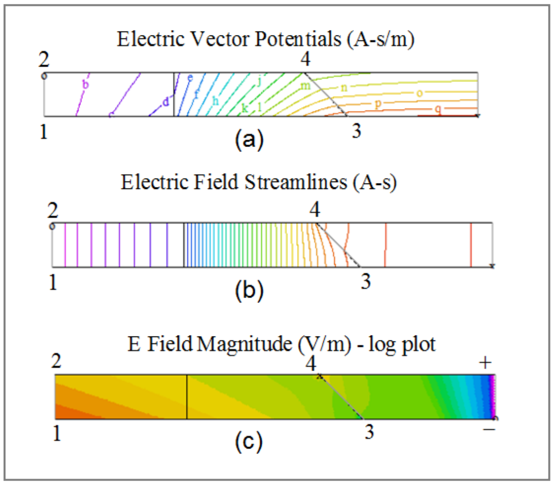

Figure 3. Layout of a) electric vector potential F, (b) Electric field streamlines E and (c) |E| in Example 1.

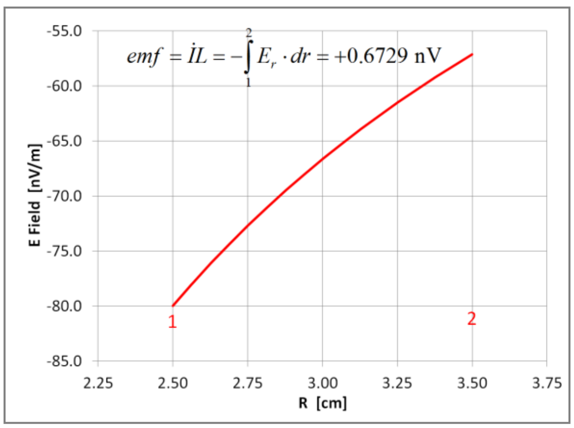

Figure 4. Drive E and *emf* in Example 1.

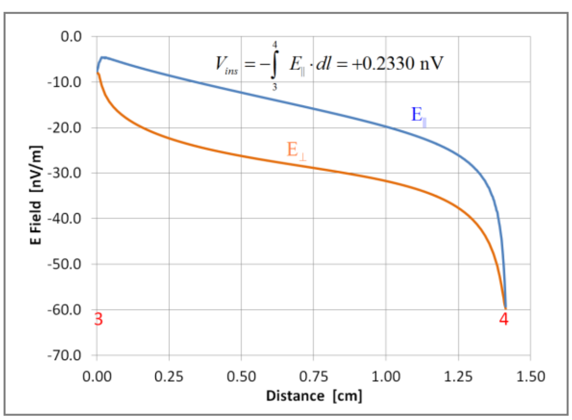

Figure 5. Monitoring E and Voltage across the insulator in Example 1.

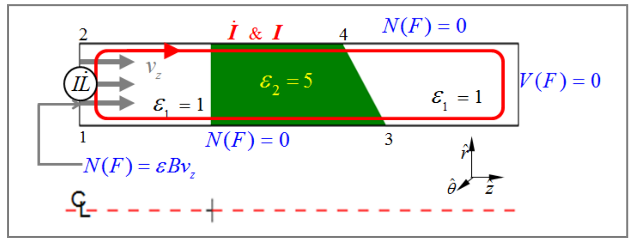

Figure 6. A coaxial geometry with a moving boundary and PEC walls; example 2.

Since the entire channel boundaries are made of PEC, the boundary condition on the remaining walls take on the *N(F) =0* form. In this simple example, there is no background magnetic field. Assuming $\dot{I}$ = -0.1 A/s, then,

$$emf = \dot{I}L = -\int_1^2 E_{\parallel} \cdot dl = +0.62729 \text{ nV} \tag{17}$$

Figure 3, 4 and 5 shows FlexPDE results for this case. The electric field stresses seen by the insulator and the voltage that develops across the insulator is of great importance in an actual design. In this made-up problem, we monitor the electric field and voltage across nodes 3 and 4 for diagnostics,

$$V_{ins} = -\int_3^4 E_{\parallel} \cdot dl = +0.2330 \text{ nV} \tag{18}$$

B. Example 2: Moving Boundary and PEC Walls

In this example, we start with the same geometry as in example 1 and add a background B field by driving a circulating current -1.0 Amp clockwise - see Figure 6. We then compress this field by forcing the left edge to move at a rate of 1.0 cm/sec in the +z direction, the moving boundary creates an $\dot{I}$ (-0.1 A/s) that satisfies the generator equation, i.e., Equation (11). The instantaneous change in inductance is,

$$\dot{L} = -\frac{L\dot{I}}{I} = -0.6729 \text{ nOhms} \tag{19}$$

The boundary condition on the left boundary is defined by,

$$N(F) = \varepsilon B v_z = \varepsilon \frac{\mu_o I}{2\pi r} v_z = -\frac{1.768\times 10^{-20}}{r} \tag{20}$$

In this case, the right hand boundary is actually a shorted boundary but is treated as an open boundary with no power flowing through it since $E_{\perp}$ =0. Note the direction of I and $\dot{I}$ are negative. The *emf* on the left boundary, between nodes 1 and 2 is,

$$emf = V_{Left} = I\dot{L} = -\int_1^2 E_r \cdot dr = +0.6729 \text{ nV} \tag{21}$$

The *emf* on the insulator edge, between nodes 3 and 4 is,

$$V_{ins} = -\int_3^4 E_{\parallel} \cdot dl = +0.2330 \text{ nV} \tag{22}$$

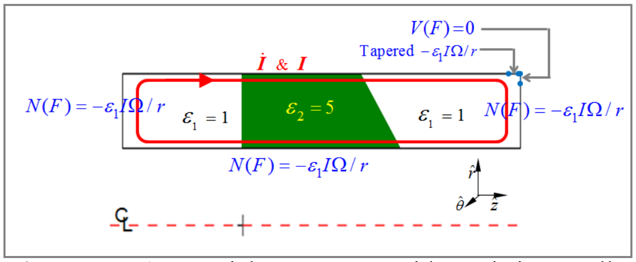

Figure 7. A coaxial geometry with resistive walls; example 3

C. *Example 3. Resistive Walls*

We start with the same geometry as in example 1 and add a background B field by driving a circulating current -1.0 amp, see Figure 7. We then allow for all the walls to be resistive. There is no moving wall in this case and the the generator equation takes on the form,

$$L\dot{I} + IR = 0 \tag{23}$$

$L\dot{I} = 0.6729$ nV as before but R has to be 0.6729 nOhms in order to satisfy Equation (23). But R is distributed among all walls – with the exception of a small region at the top right-hand side corner where resistance is tapered and *V(F) = 0* is set. This position is chosen since $E_{\perp} \approx 0$ in that proximity of the channel – see Figure 3.

We also need to choose a value of Ω to satisfy Equation (14). The value of Ω, in this example is the same for all the boundaries. In FlexPDE input deck this value is adjusted to result in R = 0.6729 nOhms for the entire channel, using $\rho = 1.7\times10^{-8}$ Ohm-m for copper,

$$\Omega = \frac{8.0939\times10^{-11}}{r} \tag{24}$$

and from Equation (13), $\delta = 2.955\times10^{-4}\,\mu$m. Therefore, the *Natural* boundary condition for each resistive boundary takes on the form,

$$N(F) = -\frac{\varepsilon_1 I\Omega}{r} = \frac{7.155\times10^{-23}}{r} \tag{25}$$

Example 4. Moving Boundary and Resistive Walls

This example is a culmination of the previous 3 examples where there is a background magnetic field that is compressed by a moving resistive boundary and is also instantaneously diffused in the neighboring resistive walls – see Figure 8. This time around, the generator equation has to be satisfied in its full form,

$$\frac{\partial(LI)}{\partial t} + IR = 0 \tag{26}$$

The boundary condition for the left moving boundary now has an added resistive component,

$$N(F) = \varepsilon B v_z - \frac{\varepsilon_1 I\Omega}{r}$$
$$= -\frac{1.768\times10^{-20}}{r} + \frac{7.155\times10^{-23}}{r} \tag{27}$$

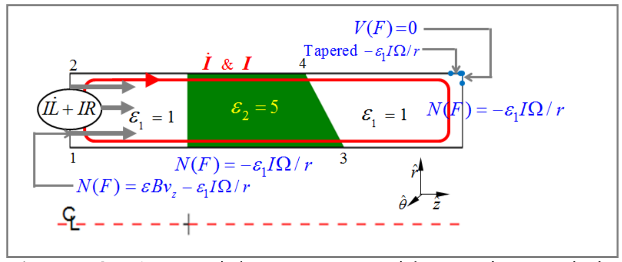

Figure 8. A coaxial geometry with moving resistive boundary and resistive walls; example 4.

At first glance, the moving wall component of Equation (27) looks to be the more dominant component but the resistive term spans over all boundaries.

In this case, the drive voltage is higher (1.96 times) in magnitude than in example 2 to compensate for the resistive losses in the generator

$$emf = V_{Left} = -\int_1^2 E_r \cdot dr = 1.3187 \text{ nV} \tag{28}$$

Consequently the voltage across nodes 3 and 4 will be higher (1.92 times) as well – not quite a linear change,

$$V_{ins} = -\int_3^4 E_{\parallel} \cdot dl = +0.4471 \text{ nV} \tag{29}$$

IV. SUMMARY

Kiuttu's method for calculation of inductive electric field using electric vector potential was implemented successfully to study magnetic flux compression in a simple coaxial configuration. FlexPDE and COMSOL were used in setting up four relevant examples. The more intricate use of *Natural* and *Value* boundary conditions was discussed and demonstrated in these examples. The details of setting up boundary conditions on PEC, resistive and moving boundaries were also explored in these examples. This formulation is a powerful, inexpensive and quick tool in design phase of coaxial generators. It can also serve as a validation tool for the more sophisticated full-wave EM codes such as COMSOL and hydro-MHD codes, such as LLNL' s ALE3D code. The method does have limitations primarily due to the quasi-static treatment used in the formulation but provides a first comprehensive approach in understanding flux compression generator physics.

V. REFERENCES

[1] G.F. Kiuttu, "Calculation of Inductive Electric Field using Electric Vector Potentials," Pulsed Power Plasma Science, Digest of Technical Papers, Vol 2, (2001)

[2] FlexPDE (v4.2) is a scripted multi-physics FEM solver of PDEs: http://www.pdesolutions.com/index.html

[3] COMSOL (v4.4) is a GUI and scriptable driven multi-physics finite element solver: http://www.comsol.com